**Chapter 5**

**Integration**

**5.6. Integrals Involving Exponential and Logarithmic Functions**

**Section Exercises**

**In the following exercises, compute each indefinite integral.**

321. 

Answer: 

323. 

Answer: 

325. 

Answer: 

327. 

Answer: 

**In the following exercises, find each indefinite integral by using appropriate substitutions.**

329. 

Answer: 

331. 

Answer: 

333. 

Answer: 

335. 

Answer: 

337. 

Answer: 

339. 

Answer: 

341. 

Answer: 

**In the following exercises, verify by differentiation that , then use appropriate changes of variables to compute the integral.**

343. 

Answer: 

345.  (*Hint:* Set .)

Answer: 

347. Write an integral to express the area under the graph of  between  and   
, and evaluate the integral.

Answer: 

**In the following exercises, use appropriate substitutions to express the trigonometric integrals in terms of compositions with logarithms.**

349. 

Answer: 

351. 

Answer: 

353. 

Answer: 

**In the following exercises, evaluate the definite integral.**

355. 

Answer: 

357. 

Answer: 

359. 

Answer: 

**In the following exercises, integrate using the indicated substitution.**

361. ; 

Answer: 

363. ; 

Answer: 

365. ; 

Answer: 

**In the following exercises, does the right-endpoint approximation overestimate or underestimate the exact area? Calculate the right endpoint estimate *R*50 and solve for the exact area.**

367. **[T]**  over 

Answer: Exact solution: , . Since *f* is decreasing, the right endpoint estimate underestimates the area.

369. **[T]**  over 

Answer: Exact solution: , . Since *f* is increasing, the right endpoint estimate overestimates the area.

371. **[T]**  over 

Answer: Exact solution: , . Since *f* is increasing, the right endpoint estimate overestimates the area (the actual area is a larger negative number).

**In the following exercises,  for . Find the area under the graph of  between the given values *a* and *b* by integrating.**

373. ; , 

Answer: 

375. ; , 

Answer: 

377. Compute the integral of  and find the smallest value of *N* such that the area under the graph  between  and  is, at most, 0.01.

Answer: . The quantity is less than 0.01 when .

379. Show that  when 

Answer: 

381. Use the previous exercise to find the antiderivative of  and evaluate 

Answer: 23

**The following exercises are intended to derive the fundamental properties of the natural log starting from the *definition*  using properties of the definite integral and making no further assumptions**.

383. Use the identity  to derive the identity 

Answer: We may assume that  Then,  Now make the substitution  so  and  and change endpoints: 

385. Use the identity  to show that  is an increasing function of *x* on  and use the previous exercises to show that the range of  is . Without any further assumptions, conclude that  has an inverse function defined on .

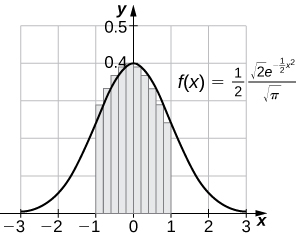
Answer: This is a proof; therefore, no answer is provided.

387. Pretend, for the moment, that we do not know that  is the inverse function of , but keep in mind that  has an inverse function defined on . Call it *E*. Show that 

Answer:  Then,  Since any number *t* can be written  for some *x*, and for such *t* we have  it follows that for any 

389. **[T]** The normal distribution in probability is given by , where *σ* is the standard deviation and *μ* is the average. The *standard normal distribution* in probability,  corresponds to  Compute the left endpoint estimates  of 

Answer: 



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